The geometrical principle on which this very simple device is based need not be proved here.* An accurate value of the true anomaly can only be found by calculation from an accurate value of E. But there may be cases in which only a rough value is required, and in any case a value of this kind, when so easily obtained, may be useful in preventing any serious slip in the calculation from being overlooked. In the case represented in the figure the value of v should be $113^{\circ}\cdot3$.

The complete instrument thus provides the means of finding, for any value of the eccentricity between 0·1 and 0·9, approximate values of the eccentric and true anomalies and of the radius vector, when the mean anomaly or time is given. It thus solves in an approximate manner the whole problem of elliptic motion as expressed by the relations (1), (2), and (3), and the ease with which this can be done suggests that the instrument may have some slight educational as well as practical value.

1906 *September* 18.

Solar Parallax Papers. No. 5.

Examination of the Photographic Places of Stars published in the Paris Eros Circulars. By Arthur R. Hinks, M.A.

1. In a preceding paper, *Solar Parallax Papers, No.* 4 (*Monthly Notices*, 1906 June, lxvi. p. 481), it was shown that the photographic right ascensions of *repère* stars, published in Paris circulars 10 and 11, had little if any magnitude equation special to each observatory (with one exception); and the conclusion was drawn that it is unlikely that they have an absolute magnitude equation common to all. This applies only to stars down to a magnitude little fainter than 9th. In the present paper I propose to extend the search for photographic magnitude equation to the declinations of the *repère* stars, and to the fainter stars in both co-ordinates; and to look for systematic errors other than those depending on magnitude, so far as they can be detected by intercomparison of published results.

2. The stars measured upon Eros plates in accordance with M. Loewy's programme are divided into three classes:—

i. The *étoiles de repère*, extending right up to the edges of plates 2° square, and generally in the outer regions rather than near the centre.

ii. The *étoiles de comparaison*, which have been used as comparison stars in visual micrometric observations. They lie mostly in a narrow belt along the track of the planet, and extend right up to two edges of the plate.

* The principle is the same as that employed in "A Method of Mechanically Compensating the Rotation of the Field of a Siderostat" (*M.N.*, vol. lxi. p. 402). See § 2 for a proof which can be adapted without difficulty to the present case, and §§ 8 and 9 for an examination of the geometry of the principle.

Table I.

Repère Stars. Magnitude Equation in Photographic Declinations.

List I. Magnitudes.	Paris minus— Alg.	Bord.	Cat.	Green.	Hels.	North.	S. Fern.	Toul.	Mean of 7.
−6·2	(5) −″·45	(2) +″·14	(3) +″·30	(3) −″·05	(3) +″·36	(3) +″·06	(5) −″·08	(5) −″·51	(24) −″·03
6·3 – 6·9	(6) − 17	...	(2) + 4	(5) + 7	(4) + 16	(5) + 22	(6) + 1	(6) − 3	(28) + 07
7·0 – 7·4	(15) − 32	(7) + 4	(9) + 12	(14) + 1	(9) − 12	(12) − 10	(15) − 10	(15) − 9	(81) − 4
7·5 – 7·9	(27) − 10	(12) + 11	(21) + 11	(20) − 1	(15) − 2	(20) − 3	(27) + 1	(27) − 7	(142) + 1
8·0 – 8·4	(60) 0	(24) + 5	(39) + 7	(47) + 3	(22) + 5	(33) 0	(53) + 13	(53) + 4	(271) + 3
8·5 – 8·8	(70) + 4	(40) 0	(43) + 3	(60) 0	(38) − 9	(46) − 2	(66) + 9	(67) + 1	(360) + 1
8·9 – 9·2	(45) + 7	(24) + 5	(31) + 13	(35) − 3	(20) − 4	(33) 0	(44) + 9	(44) + 5	(231) + 4
9·3 –	(12) + 21	(9) + 4	(8) + 11	(8) 0	(10) + 4	(7) − 3	(13) + 24	(13) + 20	(68) + 10
Totals	(240) − ·01	(118) + ·04	(156) + ·08	(192) ·00	(121) − ·02	(159) − ·01	(229) + ·05	(230) + ·01	...
List II.									
−6·2	(5) − ·23	(2) + ·56	(4) + ·53	(3) − ·09	(5) + ·14	(3) + ·40	(5) + ·26	(3) + ·43	(25) + ·30
6·3 – 6·9	(7) − 12	(4) − 11	(7) − 16	(4) − 6	(3) + 31	(1) − 19	(7) + 29	(5) + 24	(31) − 1
7·0 – 7·4	(8) − 10	(4) − 19	(5) + 16	(8) + 1	(8) + 12	(2) − 10	(8) − 5	(7) − 7	(42) 0
7·5 – 7·9	(17) + 2	(6) − 8	(12) + 3	(12) + 8	(12) − 2	(5) − 20	(17) − 4	(11) − 4	(75) − 2
8·0 – 8·4	(18) − 4	(13) − 3	(15) 0	(16) − 10	(16) + 5	(9) + 10	(25) + 2	(18) − 2	(112) 0
8·5 – 8·8	(24) + 6	(15) + 10	(16) − 8	(16) + 3	(18) + 3	(8) − 4	(24) + 5	(17) + 6	(114) + 3
8·9 – 9·2	(3) + 14	(3) − 16	(3) − 13	(3) + 6	(3) + 23	(1) + 27	(3) − 3	(3) − 12	(19) − 1
9·3	...	...	...	...	...	...	...	...	...
Totals	82) − ·01	(47) ·00	(62) + ·01	(62) − ·01	(65) + ·06	(29) + ·02	(89) + ·04	(64) − ·01	...

Note.—The number of stars contributing to each mean is given in brackets before it.

iii. The *étoiles du carré de* 20′, including all stars on the plate within a square of twenty minutes of arc, having the planet at the centre—the planet being always near the centre of the plate.

Unfortunately for the study of systematic errors, the places of the stars in these three categories have been published in three different ways in the Paris Circulars.

For the *étoiles de repère*, Tableau I. gives the mean place concluded from the whole number of plates.

For the *étoiles de comparaison*, Tableau III. gives, in some cases, collections of the separate individual results, but without indication of the plates from which they are derived; in other cases, means only.

For the *étoiles du carré* the Tableau II. gives the complete individual results, plate by plate.

It is therefore impossible to separate completely the proceeds of one plate from the rest, and this is a grave obstacle to the proper investigation of the results. For the present we are restricted to the study of mean places (except for a small central region). We shall therefore, in what follows, draw no distinction between classes II. and III., and shall refer to all the fainter stars as "comparison stars."

3. *Magnitude Equation in Photographic Declinations. Repère Stars.*—The comparison of the photographic declinations of *repère* stars, grouped according to magnitude, is presented in the same form as was the comparison of photographic R.A.'s in my last paper (*loc. cit.*, p. 483), but with the addition of a column for Paris minus mean of seven (Algiers being excluded).

For list I. stars, Algiers has a considerable magnitude equation in declination, as it had in R.A. San Fernando shows no very evident sign of it; but Toulouse has a well-marked magnitude equation, which is unexpected and very interesting. Bordeaux, Catania, Greenwich, Helsingfors, Northfield, and Paris show no serious divergence one from another, and we conclude that they are sensibly free from magnitude equation.

The comparison with list II. extends at present only to R.A. $3^h\ 17^m$ (corresponding to 1901 January 25). Up to that point there is little evidence of magnitude equation, even for Algiers.

4. *Magnitude Equation in Visual Declinations. Tucker's System.*—This comparison follows very closely on the lines of the preceding.

Comparison with the mean of six observatories (excluding Algiers, Toulouse, and San Fernando), and with the six individually, shows little or no evidence of any magnitude equation in Tucker's declinations, unless possibly for the brightest stars, for which material is insufficient.

And the various systems adopted by different observatories show no certain trace of relative magnitude equation when compared with Tucker.

We may conclude that the meridian circle declinations in general are practically free from personality depending on magnitude —an interesting and somewhat unexpected conclusion.

Table II.

Repère Stars. Examination of Tucker's Declinations for Magnitude Equation.

	System T. minus—									
List I.	**Alg.**	**Bord.**	**Cat.**	**Green.**	**Hels.**	**North.**	**Paris.**	**S. Fern.**	**Toul.**	**Mean of 6.**
−6·2	(7) −″·42	(2) +″·18	(5) −″·07	(4) +″·13	(4) +″·32	(5) +″·22	(5) +″·12	(7) −″·06	(7) −″·33	(25) +″·14
6·3 – 6·9	(7) − 17	…	(2) + 5	(5) + 11	(4) + 21	(5) + 27	(6) + 7	(7) + 12	(7) + 9	(22) + 15
7·0 – 7·4	(20) − 23	(7) + 7	(13) + 10	(18) + 7	(12) 0	(14) + 3	(15) + 6	(20) − 4	(20) − 7	(79) + 6
7·5 – 7·9	(35) − 6	(12) + 8	(25) + 18	(22) + 2	(17) 0	(28) − 2	(27) − 2	(35) − 4	(35) − 4	(131) + 4
8·0 – 8·4	(58) 0	(24) + 2	(44) + 6	(49) + 2	(26) + 4	(36) − 1	(53) − 2	(58) + 12	(57) + 1	(232) + 2
8·5 – 8·8	(74) + 6	(41) + 3	(51) 0	(60) + 1	(39) − 1	(47) + 3	(66) + 2	(77) + 12	(77) + 2	(304) + 1
8·9 – 9·2	(54) + 10	(24) + 2	(38) + 9	(38) 0	(23) − 9	(35) − 6	(44) − 2	(55) + 9	(55) + 3	(202) − 1
9·3 –	(12) + 13	(9) + 4	(9) + 17	(8) + 2	(10) − 1	(8) − 1	(13) − 4	(14) + 17	(14) + 16	(57) + 2
Totals	(267) ·00	(119) + ·04	(187) + ·07	(204) + ·02	(135) ·00	(178) + ·01	(229) ·00	(273) + ·08	(272) + ·01	…
List II.										
−6·2	(5) − ·51	(2) − ·08	(4) + ·10	(3) + ·09	(5) + ·06	(3) + ·08	(5) − ·28	(5) − ·02	(3) − ·18	(22) − ·01
6·3 – 6·9	(7) − 25	(4) − 25	(7) − 25	(4) − 20	(3) + 3	(1) − 75	(7) − 13	(7) + 16	(5) − 39	(26) − 20
7·0 – 7·4	(9) − 17	(4) − 21	(6) + 17	(9) − 5	(8) + 2	(2) − 14	(8) − 5	(8) − 12	(7) − 14	(37) − 2
7·5 – 7·9	(17) − 8	(6) − 23	(12) − 10	(12) + 1	(12) − 6	(5) − 23	(17) − 14	(17) − 18	(11) − 12	(64) − 11
8·0 – 8·4	(18) − 20	(13) − 21	(15) − 15	(16) − 22	(16) + 2	(9) − 2	(19) − 16	(19) − 13	(18) − 16	(88) − 13
8·5 – 8·8	(24) 0	(15) 0	(16) − 15	(16) − 4	(19) − 3	(8) − 7	(24) − 6	(24) − 1	(17) − 2	(98) − 6
8·9 – 9·2	(3) − 13	(3) − 43	(3) − 41	(3) − 21	(3) − 1	(1) + 3	(3) − 27	(3) − 30	(3) − 39	(16) − 23
9·3 –	…	…	…	…	…	…	…	…	…	…
Totals	(83) − ·13	(47) − ·09	(63) − ·12	(63) − ·09	(66) − ·01	(29) − ·09	(83) − ·13	(83) − ·08	(64) − ·14	…

But the comparison of Tucker with the photographic results of Algiers and Toulouse exhibits the magnitude equation in the latter previously discovered.

The results of the comparison with list II. are too irregular to be of much service.

5. *Magnitude Equation in Photographic R.A. Comparison Stars.*—In extending to the fainter stars our search for magnitude equation, we meet with the difficulty that the magnitudes assigned to these stars at different observatories are discordant. For example, on tabulating several hundred comparison stars, we find that the magnitudes assigned at Paris are greater in the mean by $1^m{\cdot}6$ than those assigned to the same stars at Bordeaux, and greater by $0^m{\cdot}6$ than those assigned to the same stars on Catania plates, though these were measured at Paris. It would seem that the Paris "magnitudes" are really intensities of image on the plate, unreduced to any photometric scale; excellent, therefore, for investigating magnitude equation on plates treated individually, but of less merit when combined into means.

Because of these discordances in the assigned magnitudes, one must discuss the differences—Paris—Bordeaux, for example—in duplicate, once with the Paris magnitudes, and a second time with the Bordeaux. We find below the classification according to magnitude of the differences Paris—Bordeaux, and also of the differences Bordeaux—Paris (the same quantities with signs changed). The first classification is by Paris magnitudes, and the compartment $9^m{\cdot}9$–$10^m{\cdot}5$ contains 17 stars. The second division is by Bordeaux magnitudes, and the same compartment contains 101 stars. A result still more curious occurs in the comparison between Paris and San Fernando. The last group $12^m{\cdot}2$ and fainter contains 158 stars in the first classification by Paris magnitudes. On repeating it with the San Fernando magnitudes, all but two of these stars are thrown back into preceding groups. It is clear that there will be difficulties in expressing an error as a function of the magnitude.

6. The results here discussed cover nearly the same stretch of the planet's path as does list I. of the *repère* stars, and depend largely, though not entirely, upon stars common to the Paris and other series. The comparison is therefore somewhat incomplete. Quite early in the work it was realised that definitive corrections could not be derived from this material as it stands; that we should have to repeat the whole work in greater detail, and that it was not worth while to spend time in making the present preliminary discussion absolutely complete.

Photographic places of comparison stars have been published in Circulars 10 and 11 by Paris, Bordeaux, Catania, San Fernando, Toulouse, and Algiers. Each one of these six observatories has been compared with each of the others. But it is hardly necessary to give here all the results. It will be sufficient if we give the comparisons of Paris, San Fernando, and Algiers with one another and with the others.

The first set shows that there is no serious discordance between Paris and others, with the exception of San Fernando. To see if the apparent magnitude equation in San Fernando is real, we compare it with each of the others. They are unanimous in showing a relative magnitude equation, approximately linear, of about $0^s.009$ per magnitude. But we must notice that this is of opposite sign to that suspected in the *repère* stars. We shall return to this point in § 8.

The comparison of others with Algiers shows that the extraordinary magnitude equation of the *repère* stars is maintained through the fainter magnitudes. For stars of the sixth magnitude the Algiers R.A. is small by about $0^s.03$; for the twelfth magnitude it is large by $0^s.08$—the whole range equal to about $1''$.

Table III.

Comparison Stars. Magnitude Equation in Photographic R.A.'s.

Paris minus—

m	Bord. s	Cat. s	S. Fern. s	Toul. s	Alg. s
– 9·2	(9) – ·001	(14) + ·016	(20) + ·009	(21) + ·001	(20) – ·005
9·3 – 9·8	(8) – 10	(14) – 3	(14) – 5	(17) – 9	(16) – 33
9·9 – 10·5	(17) – 2	(24) + 6	(46) + 9	(50) + 6	(42) – 32
10·6 – 11·2	(36) + 1	(73) + 2	(78) + 12	(79) 0	(71) – 48
11·3 – 12·1	(56) – 1	(153) – 4	(139) + 22	(129) + 6	(105) – 67
12·2 –	(93) – 7	(153) + 2	(158) + 21	(148) 0	(80) – 88
Totals	(219) – ·004	(431) + ·001	(455) + ·017	(444) + ·002	(344) – ·056

San Fernando minus—

m	Bord. s	Cat. s	Paris. s	Toul. s	Alg. s
– 9·2	(28) – ·007	(50) – ·001	(91) – ·008	(98) – ·004	(93) – ·031
9·3 – 9·8	(39) – 12	(50) – 16	(68) – 14	(68) – 8	(61) – 68
9·9 – 10·5	(72) – 20	(112) – 21	(113) – 17	(109) – 15	(101) – 86
10·6 – 11·2	(57) – 23	(112) – 24	(135) – 21	(96) – 12	(74) – 92
11·3 – 12·1	(16) – 39	(32) – 27	(46) – 34	(38) – 24	(24) – 108
12 2 –	(1) + 20	(1) + 40	(2) – 39	(4) + 2	(3) – 138
Totals	(213) – ·019	(357) – ·019	(455) – ·017	(413) – ·011	(356) – ·072

Algiers minus—

m	Bord. s	Cat. s	Paris. s	S. Fern. s	Toul. s
– 9·2	(25) + ·023	(41) + ·031	(76) + ·022	(79) + ·032	(81) + ·026
9·3 – 9·8	(31) + 57	(40) + 63	(63) + 50	(59) + 60	(63) + 54
9·9 – 10·5	(26) + 63	(37) + 83	(40) + 62	(46) + 66	(45) + 57
10·6 – 11·2	(42) + 66	(72) + 79	(76) + 69	(86) + 95	(84) + 82
11·3 – 12·1	(39) + 73	(39) + 87	(81) + 77	(80) + 96	(85) + 82
12·2 –	(2) + 30	(5) + 112	(8) + 89	(6) + 90	(10) + 88
Totals	(155) + ·062	(234) + ·071	(344) + ·056	(356) + ·072	(368) + ·062

Tables similar to the above for Bordeaux, Catania, and Toulouse show few features of interest. For Bordeaux minus others, the numbers fall slightly to the third group, and rise again. For Catania and Toulouse there is a slight upward tendency at the beginning and end. But the whole range is small, equivalent to about $0''.2$; and, in light of what will follow, we cannot attribute it to magnitude equation.

7. *Magnitude Equation in Photographic Declinations. Comparison Stars.*—The procedure followed in discussing the R.A.'s has been repeated for the declinations without change. A selection of the more interesting results is given in Table IV.

Paris, Bordeaux, and Catania agree fairly well. We can hardly look upon the differences Paris – Bordeaux as significant of magnitude equation until we have examined the effects of the difference of adopted places in Bordeaux, which is a very incomplete series.

Toulouse diverges strongly from these three accordant series, and has a well-marked nearly linear magnitude equation in the declinations of comparison stars of about $0''.1$ per magnitude.

San Fernando is a difficult case. We may summarise thus:—

Paris —San Fernando (P. magnitudes) shows a very slight M.E.
Bordeaux—San Fernando (B. magnitudes) shows decided M.E.
Catania —San Fernando (C. magnitudes) shows large M.E. of more than $0''.1$ per magnitude.

On the other hand:

San Fernando—Paris (S.F. magnitudes) shows no M.E. whatever.
,, —Bordeaux (S.F. magnitudes) shows none, or, at most, traces of a non-linear M.E.
,, —Catania (S.F. magnitudes) shows a very small and uncertain M.E.

The discordance between the comparisons Catania—San Fernando and San Fernando – Catania, that is to say, between the figures grouped differently according as one accepts the Catania or San Fernando magnitudes, shows that no definitive corrections can be derived from the present work. The magnitude equation, if any, is doubtless entangled with other systematic errors of Catania or San Fernando, for which see later, § 14.

Algiers shows a considerable magnitude equation, as before, but it is not nearly so variable in the fainter magnitudes as in the brighter.

Table IV.

Comparison Stars. Magnitude Equation in Photographic Declinations.

Paris minus—

	Bord.	Cat.	S. Fern.	Toul.	Alg.
– 9·2	(8) + ″·03	(14) – ″·03	(21) – ″·09	(21) + ″·11	(20) – ″·04
9·3 – 9·8	(5) 0	(14) – 5	(14) + 22	(19) + 28	(18) + 22
9·9 – 10·5	(18) + 2	(25) + 1	(47) + 24	(48) + 28	(42) + 33
10·6 – 11·2	(37) + 9	(72) + 3	(78) + 20	(81) + 29	(68) + 29
11·3 – 12·1	(59) + 12	(153) + 2	(138) + 30	(126) + 37	(103) + 39
12·2 –	(93) + 4	(152) + 1	(157) + 28	(149) + 45	(93) + 46
Totals	(220) + ·07	(430) + ·01	(455) + ·26	(444) + ·36	(344) + ·35

San Fernando minus—

	Bord.	Cat.	Paris.	Toul.	Alg.
– 9·2	(29) – ″·10	(50) – ″·15	(93) – ″·24	(97) – ″·05	(93) – ″·03
9·3 – 9·8	(38) – 13	(52) – 13	(67) – 25	(69) + 4	(61) + 11
9·9 – 10·5	(73) – 24	(111) – 25	(119) – 23	(108) + 14	(101) + 18
10·6 – 11·2	(59) – 17	(113) – 29	(129) – 30	(98) + 16	(74) + 23
11·3 – 12·1	(15) – 16	(31) – 24	(45) – 25	(38) + 7	(24) + 25
12·2 –	(1) – 20	(1) – 10	(2) + 7	(4) + 7	(3) + 4
Totals	(215) – ·18	(358) – ·23	(455) – ·26	(414) + ·08	(356) + ·13

Toulouse minus—

	Bord.	Cat.	Paris.	S. Fern.	Alg.
– 9·2	(21) – ″·11	(45) – ″·20	(82) – ″·22	(74) + ″·05	(77) ″·00
9·3 – 9·8	(32) – 18	(44) – 21	(74) – 31	(70) – 5	(65) + 8
9·9 – 10·5	(72) – 32	(129) – 29	(129) – 35	(127) – 16	(112) – 1
10·6 – 11·2	(55) – 35	(98) – 35	(123) – 42	(113) – 8	(94) + 7
11·3 – 12·1	(12) – 61	(15) – 28	(36) – 56	(30) – 9	(20) 0
12·2 –	...	...	...	...	...
Totals	(192) – ·30	(331) – ·29	(444) – ·36	(414) – ·08	(368) + ·03

8. *Summary of Results for Photographic Magnitude Equation.* —We may now bring together the results for the *repère* and comparison stars.

Regrouping the *repère* stars into larger magnitude groups, to avoid the roughness due to paucity of numbers, we have the following table for the comparison of Toulouse, Algiers, and San Fernando, in which magnitude equation is discovered or suspected, with the mean of Paris, Catania, Bordeaux, in which it does not appear.

Table V.

Collected Results for Magnitude Equation.

R.A.	Toulouse minus Mean of P.C.B.	San Fernando minus Mean of P.C.B.	Algiers minus Mean of P.C.B.
m	s	s	s
− 7·4	+0·001	−0·015	−0·031
7·5 − 8·4	0	− 10	− 6
8·5 − 9·2	+ 1 − 0·001	− 2 − 0·005	+ 14 + 0·025
9·3 − 9·8	5 + 2	+ 10 − 14	+ 23 + 57
9·9 − 10·5	0	− 19	+ 69
10·6 − 11·2	− 7	− 23	+ 73
11·3 − 12·1	− 6	− 33	+ 79
12·2 −	...	...	+ 77
Decl.			
− 7·4	+0″·22	+0″·14	+0″·38
7·5 − 8·4	+ 5	− 4	+ 8
8·5 − 9·2	0 − 0″·18	− 6 − 0″·16	− 2 − 0″·16
9·3 − 9·8	− 15 − 23	− 18 − 17	− 16 − 24
9·9 − 10·5	− 32	− 24	− 28
10·6 − 11·2	− 37	− 25	− 38
11·3 − 12·1	− 48	− 22	− 44
12·2 −	...	...	− 35

The first of the two columns belonging to each observatory is derived from the *repère* stars, the second from the comparison stars. The roughness at the points where these overlap need not be taken too seriously, for the magnitudes of the first are Tucker's magnitudes, *i.e.* practically B.D. magnitudes, while those of the second are photographic magnitudes.

The conclusions, in brief, are as follows:—

Toulouse has very little magnitude equation in R.A., but a large one in declination, which is nearly linear.

Algiers has a very large magnitude equation in both co-ordinates; larger in R.A. than in Decl.; not strictly linear in either, but altering little for the faintest stars.

San Fernando appears to have a fairly small and decidedly non-linear magnitude equation in both co-ordinates.

In estimating the weight of these conclusions we must remember that there are other systematic errors involved with the magnitude equation, which have not yet been discussed; and that the actual quantity upon which magnitude equation must depend is likely to be the number representing the intensity of the image upon the plate, rather than the number more or less reduced to photometric scale which is derived from it and published as the photographic magnitude (except apparently at Paris). For these reasons no

attempt will be made just yet to determine a set of definitive corrections.

9. For the same reasons it is premature to attempt to settle the question—What is the cause of these errors? But we may note in passing, that at Toulouse and at Algiers the photographic equatorial is mounted in the "English" style,* so that errors of the objective are not reversed on crossing the meridian; while the plates are reversed during measurement. One will in this case naturally look first for the error in the objective. At San Fernando the plates are not reversed during measurement, and the small non-linear magnitude equation may very well be due to this cause.

10. *Progressive Discordances. Repère Stars.*—If a photographic telescope gives results which are systematically wrong over a long series, there is naturally a strong probability that the cause is instrumental; and if so, the effect will most likely be a function of the magnitude. We have therefore dealt with magnitude equation first.

We have now to look for discordances of a semi-systematic character, varying from point to point along the path of the planet, —such discordances, for example, as might be due to roughness in the adopted places of the *repère* stars. We will look for them first in the concluded photographic places of the *repère* stars themselves.

Each *repère* star has been assigned to one or other of a series of groups on centres 2° apart along the orbit of the planet (alternate centres of list given in Paris Circular No. 3, p. 7, for the "special series" plates).

11. Consider in the first place the observatories which have reduced to a common fundamental system (that of M. Loewy), viz. Catania, Greenwich, Paris, and Toulouse. The group means of the differences of these series are given in the table on the next page.

In studying this table we must remember—

(*a*) That the Greenwich plates were reduced first of all to an independent system, and afterwards mean corrections were applied to each plate to reduce to Loewy's system.

(*b*) That the Toulouse system is not exactly the same as Loewy's, though for present purposes the difference is insignificant.

(*c*) That the plates taken at Catania were measured and reduced at Paris, and ought to be particularly concordant with the Paris plates.

(*d*) That the Tolouse declinations are slightly affected by magnitude equation.

(*e*) That stars appearing in the same group are not necessarily reduced with the same selection of *repère* stars, or even with any single star common to the two reductions.

* I am indebted for this information to the directors of the observatories in question. It is not a little curious that, although the astrographic telescopes have now been at work for fifteen years, one cannot always find the slightest account of the type of mounting employed. Yet the question whether or not the objective is reversed on crossing the meridian is fundamental in all inquiries about systematic error.

(*f*) On the other hand, that the mean photographic place of one star will often depend on a considerable range of *repère* stars grouped on different centres, and that the errors of individual adopted places will be smoothed out much more effectually than is the case on Astrographic Catalogue plates, with their centres at uniform distances.

Bearing these facts in mind, we may draw the following conclusions from Table VI. :—

(1) The systematic difference between Paris and Greenwich, taken over the whole series, is quite insensible. And we found that there was no trace of relative magnitude equation between

Table VI.

Repère Stars reduced to Loewy's System. Progressive Discordances.

Centre.	Date. 1900.	Paris minus— Greenwich.	Toulouse.	Catania.
		s ″	s ″	s ″
6	Oct. 2	(11) ·000 + ·03	(11) + ·005 + ·15	(11) − ·001 + ·02
8	,, 8	(15) + 3 − 2	(17) + 4 − 1	(17) + 3 0
10	,, 14	(11) − 13 + 4	(15) − 7 + 8	(13) + 22 + 20
12	,, 19	(15) 0 − 3	(20) 0 0	(14) − 3 + 3
14	,, 25	(11) + 1 0	(13) + 6 + 4	(12) + 26 + 6
16	Nov. 1	(14) + 13 + 1	(17) + 3 − 1	(16) + 11 + 10
18	,, 7	(16) − 9 0	(21) + 2 − 1	(21) − 6 + 6
20	,, 14	(16) + 3 − 1	(19) + 16 + 7	(13) + 2 + 1
22	,, 21	(16) − 7 + 4	(18) − 4 + 4	(12) − 8 + 18
24	,, 27	(15) + 3 − 1	(16) + 5 + 3	(13) + 17 + 21
26	Dec. 3	(7) + 4 − 21	(15) 0 + 2	(9) + 21 − 3
28	,, 9	(11) + 5 + 4	(11) + 2 0	(3) + 46 − 41
30	,, 14	(6) + 3 + 5	(7) + 15 − 20	(5) − 1 − 5
32	,, 18	(14) + 4 + 6	(14) + 4 + 3	(6) + 21 + 44
34	,, 22	(16) − 5 + 7	(16) + 1 + 3	...
36	,, 26	(16) − 4 + 2	(17) + 3 − 5	(1) − 18 + 40
Average without regard to sign		s ″ ·0048 ·040	s ″ ·0048 ·048	s ″ ·0137* ·147*
Mean		·0000 + ·028	+ ·0033 + ·072	+ ·0060 + ·086

Note.—The averages are formed from the group means given above, and sometimes the same star occurs in two groups. These duplicates have been expunged before the final means were taken, in which no star is reckoned more than once.

them. We may provisionally adopt the figures in the columns Paris—Greenwich as indicative of the degree of concordance that

* Or, omitting the last four, which depend on few observations, the averages are 0·0109 0″·082.

may be reached by two series made independently with different instruments and reduced to the same standard.

(2) The divergences from centre to centre seem to be due more to the roughness of the *repère* star places than to errors of measurement, or real errors of the photographs. This may be seen in the columns for Paris—Toulouse. The Toulouse plates are few in number compared with Greenwich and Paris. But the average discordance of the groups is nearly the same in Paris—Toulouse as it is in Paris—Greenwich.

(3) The Catania plates are still fewer in number, being practically only alternate plates of the special series. Although they were measured and reduced at Paris, the discordances from Paris are large. At first sight this might be attributed to small number of plates and want of averaging out errors by overlapping. But the discordances seem to be too large for any such simple explanation; notice especially centres 10, 14, 24, and 26. We will postpone discussion of them till we come to deal with the fainter stars.

Table VII.

Repère Stars not reduced to Loewy's System. Progressive Discordance from Paris of Photographic and Adopted R.A.'s.

		Paris minus—								
		San Fernando.			Northfield.			Helsingfors.		
Centre.	Date. 1900.	Phot.		Adopt.	Phot.		Adopt.	Phot.		Adopt.
		s		s	s		s	s		s
6	Oct. 2	(11)	+·004	+·009	...		...	...		...
8	,, 8	(17)	+13	+6	(13)	+·019	+·005	...		...
10	,, 14	(15)	+3	+9	(14)	−1	+8	...		...
12	,, 19	(20)	+10	+10	(15)	+12	+9	(8)	+·008	+·010
14	,, 25	(13)	+9	+11	(10)	+10	+5	(9)	+32	+10
16	Nov. 1	(17)	+11	+15	(11)	0	+10	...		...
18	,, 7	(21)	+15	+15	(14)	−1	+11	(18)	−3	+9
20	,, 14	(19)	+21	+9	(11)	+3	+7	(4)	−13	+10
22	,, 21	(18)	+13	+11	(10)	−3	+6	(13)	+7	+11
24	,, 27	(16)	+11	+11	(14)	+16	+8	(15)	+22	+17
26	Dec. 3	(15)	−1	0	(11)	+13	+11	(13)	+25	+14
28	,, 9	(11)	+5	+8	(8)	+2	+8	(12)	+18	+13
30	,, 14	(7)	+5	+3	...		...	...		...
32	,, 18	(14)	+5	−1	(11)	+3	+7	...		...
34	,, 22	(16)	−3	+5	(14)	+4	+4	(11)	+9	+10
36	,, 26	(17)	+11	+3	(11)	+3	+6	(12)	+15	+8
Mean			+·0082	+·0078		+·0057	+·0075		+·0120	+·0112

(4) Briefly, then, we conclude from Table VI. that it is possible to get consistent places, both R.A.'s and declinations, from two photographic series made at different observatories—witness Paris

and Greenwich; but even under conditions which should make for accordance, one finds discordances unexpectedly big, as in the case of Catania.

12. So far we have dealt with series reduced to the same fundamental system. We must now examine how far difference of adopted system is likely to produce discordance.

We will take as examples San Fernando, Northfield, and Helsingfors. The comparison of R.A.'s, both deduced and adopted, is given in Table VII.

It will be seen at once that the difference in the adopted places is responsible for the principal part of the difference in the concluded photographic places. At San Fernando and Helsingfors the mean difference is exactly accounted for. At Northfield the agreement is less satisfactory, but the series is less complete.

13. The declinations have been treated in the same way, and give a very similar result. The mean discordances of the photographic declinations are due almost entirely to the difference of adopted places.

I think we may conclude that the policy of allowing any observatory to make its own system of adopted places for the *repère* stars has led to well-marked systematic discordances between some of the photographic series, of the order of $0^s.01$ and $0''.1$, which it will be worth while to eliminate.

14. *Comparison Stars. Progressive Discordances.*—We have now to examine how far the photographic places of the fainter stars differ from centre to centre. Our material is the same as that discussed above for magnitude equation. Each observatory has been compared with each of the others. The results have been studied by plotting in curves coloured to indicate the observatory of origin. These cannot be reproduced, and the tables which represent them numerically are not quite successful in showing up the features which are conspicuous in the diagrams. We can give only a few examples, and a general statement of results.

A typical section is that traversed by the planet between October 16 and November 4, which has been divided into blocks lettered K, L, M, N, O. Table VIII. gives the R.A. comparisons.

Since all these comparisons are differential, it is impossible to obtain the actual error of any series. But we sometimes can get a very good idea of which is in the wrong. Take, for instance, the comparison of Catania with others. In block K the Catania R.A.'s are about $0^s.04$ greater than all the others; in block L they nearly agree; in block M they are decidedly less; in block N they have risen again; and fall once more in O as compared with the others. I think that we may conclude in this case that the irregularity is mainly in the Catania observations.

In this way it has been found possible to obtain a general idea of the degree of systematic error in the photographic places, both R.A.'s and declinations, of the comparison stars. The result is by no means reassuring. Let us take the observatories in order.

Table VIII.

Comparison Stars. Photographic R.A. Progressive Discordances between Results from Different Observatories.

	Paris –	Bord. –	Cat. –	S. Fern. –	Toul. –
K. Oct. 16–19.	s	s	s	s	s
Paris	...	(14) + ·012	(29) + ·040	(14) – ·021	(12) + ·001
Bord.	(14) – ·012	...	(15) + 34	(13) – 20	(12) – 7
Cat.	(29) – 40	(15) – 34	...	(12) – 54	(9) – 37
S. Fern.	(14) + 21	(13) + 20	(12) + 54	...	(13) + 21
Toul.	(12) – 1	(12) + 7	(9) + 37	(13) – 21	...
L. Oct. 19–23.					
Paris	...	(28) + ·009	(16) – ·002	(19) – ·003	(26) – ·003
Bord.	(28) – ·009	...	(10) – 5	(17) – 10	(24) – 12
Cat.	(16) + 2	(10) + 5	...	(6) – 13	(8) – 12
S. Fern.	(19) + 3	(17) + 10	(6) + 13	...	(20) – 4
Toul.	(26) + 3	(24) + 12	(8) + 12	(20) + 4	...
M. Oct. 23–27.					
Paris	...	(29) – ·002	(36) – ·030	(39) – ·008	(32) + ·011
Bord.	(29) + ·002	...	(36) – 22	(36) – 29	(23) + 15
Cat.	(36) + 30	(36) + 22	...	(37) + 3	(23) + 39
S. Fern.	(39) + 8	(36) + 29	(37) – 3	...	(28) + 28
Toul.	(32) – 11	(23) – 15	(23) – 39	(28) – 28	...
N. Oct. 27–31.					
Paris	...	(31) – ·003	(38) – ·003	(32) – ·033	(19) – ·023
Bord.	(31) + ·003	...	(35) + 10	(35) – 38	(18) – 5
Cat.	(38) + 3	(35) – 10	...	(34) – 41	(21) – 19
S. Fern.	(32) + 33	(35) + 38	(34) + 41	...	(20) + 32
Toul.	(19) + 23	(18) + 5	(21) + 19	(20) – 32	...
O. Nov. 1–4.					
Paris	...	(20) – ·016	(33) – ·026	(21) – ·002	(22) – ·031
Bord.	(20) + ·016	...	(28) – 13	(21) + 21	(21) – 2
Cat.	(33) + 26	(28) + 13	...	(22) + 30	(24) + 7
S. Fern.	(21) + 2	(21) – 21	(22) – 30	...	(22) – 21
Toul.	(22) + 31	(21) + 2	(24) – 7	(22) + 21	...

15. Paris comes out decidedly the best. Only in a few cases is Paris systematically different from all the others.

Bordeaux results are also good. The differences from other observatories are very irregular, and to be attributed in the main to the others, and not to Bordeaux.

Catania is an extremely interesting case. The discordances from Paris are quite extraordinary. The following table gives their values in the section where they are best determined:—

Table IX.

Comparison Stars. Progressive Discordances.
Paris minus Catania.

Block.	No. of Stars.	Mean Discordance in R.A.	Decl.
		s	″
E.	(8)	− ·013	− ·06
F.	(32)	− 18	− 5
G.	(12)	+ 32	− 9
H.	(30)	− 21	+ 24
K.	(29)	− 40	+ 12
L.	(16)	− 2	− 12
M.	(36)	+ 30	+ 12
N.	(38)	+ 3	− 19
O.	(33)	+ 26	− 7
P.	(39)	+ 2	+ 7
Q.	(41)	− 26	− 13
R.	(38)	+ 21	0
S.	(4)	0	+ 13
T.	(11)	+ 34	− 11
U.	(11)	− 40	+ 11
V.	(2)	− 25	− 5
X.	(7)	+ 74	− 2

The greater part of the error is certainly in the Catania places. It is less in Decl. than in R.A. It does not correspond at all well with the rather large discordance between Paris and Catania for the *repère* stars, and cannot be due to roughness in the adopted places of the stars, for the places where it is greatest are generally those where Paris plates are most numerous.

Up to the present I have not succeeded in determining the form of this error, or in finding any explanation of it. That two series of plates, taken indeed with different telescopes, but measured and reduced by the same hands, should give results so remarkably discordant, could hardly have been expected by the most avowed disbeliever in the accuracy which is claimed, and generally with truth, for the photographic methods.

Toulouse and San Fernando exhibit great and very variable divergences from the others; but we have already shown that these series are affected by rather large magnitude equations, which in a climate of variable transparency must necessarily be associated with semi-systematic mean errors of the kind we are

now considering. The only sure method of procedure in such a case seems to be a discussion for magnitude equation plate by plate, which is impossible unless the published material is amplified.

16. *Conclusion.*—The results of this discussion make it clear that the star places published in the Paris Circulars are by no means homogeneous, and that errors exist in some of the series which, uneliminated, would ruin the determination of the solar parallax. I have, however, full confidence that these difficulties can be overcome. By the great kindness of the directors of several observatories, the separate results from each of their plates have recently been placed at my disposal, and the discussion of this material is now proceeding at Cambridge, with results which I hope to communicate to the Society.

17. But our conclusions have, it seems to me, an interest wider than that of the particular problem in hand. Most of the photographic telescopes concerned have been engaged for years upon the Astrographic Chart and Catalogue. With that experience behind them, they undertook shares in the Eros co-operation; and several of the resulting series of star places are affected by errors much larger than are accounted tolerable in the Astrographic Catalogue—a disquieting result.

18. The work summarised in this paper was done concurrently with that described in paper No. 4, and the same acknowledgments are due to the Government Grant Fund of the Royal Society; and to Miss Julia Bell, who has carried out the greater part of the computation.

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On the Distribution of Energy in the Continuous Spectrum.

By E. T. Whittaker, Sc.D., F.R.S., Royal Astronomer of Ireland.

The distribution of energy in the continuous spectrum of the "black body" at various temperatures, which is of interest astronomically from its application to the problem of solar and stellar temperatures, has been studied from the experimental side by many physicists, and the results obtained have been co-ordinated into the form of empirical laws of radiation. One such empirical law, which is due to Lummer and Pringsheim,* and closely represents the observations, is to the effect that the intensity in the part of the spectrum at wave-length λ, radiated by a black body at the absolute temperature T, is proportional to

$$T\lambda^{-4}e^{-\frac{c}{(\lambda T)^{1\cdot 3}}} \quad . \quad . \quad . \quad . \quad (I.)$$

where c is a constant.

The mode of occurrence of T in this formula is not really empirical, as it can be proved by thermodynamical considerations

* *Deutsch. Phys. Gesells. Verh.*, 2, 12*a*, 103–180 (1900).